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Development and Validation of PESCAD: An Inductive Approach to High School Mathematics Teaching

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Abstract

This study focused on proposing a teaching approach called PESCAD (Problem, Examination, Solution, Checking, Appeals, Discussion), which follows an inductive method. It involved developing a sample lesson and examining how experts and professionals would assess it. The project aimed to delve into the analysis, design, and development of the lesson exemplar incorporating PESCAD; the effectiveness of PESCAD in fostering 21st-century skills; the efficacy of PESCAD in promoting higher-order thinking skills; how Mathematics educators would assess PESCAD based on reliability, effectiveness, feasibility, inclusivity, and flexibility; and the feedback and recommendations from the validators to enhance PESCAD further.

PESCAD, as an inductive mathematics teaching method, effectively fosters the development of 21st-century skills and higher-order thinking skills in students. By shifting the focus from rote memorization to student-driven discovery, PESCAD enhances essential competencies such as problem-solving, critical thinking, and intellectual autonomy, which are crucial for success in today's academic and professional landscapes. The structured steps of PESCAD—Problem, Examination, Solution, Checking, Appeals, and Discussion—facilitate an engaging and interactive learning environment, promoting active student participation and a deeper understanding of complex mathematical concepts.

This research employed a developmental approach, where the researcher constructed a sample mathematics lesson to represent the inductive teaching method. Six mathematics teachers from Nueva Ecija High School validated the lesson. The findings indicate that PESCAD has the potential to enhance 21st-century skills and higher-order thinking skills among students. Validation from mathematics teachers highlights PESCAD's reliability, effectiveness, feasibility, inclusivity, and flexibility, particularly in addressing the limitations of traditional deductive teaching methods.

Unlike classical approaches, PESCAD uniquely combines reliability, flexibility, and inclusion while enhancing 21st-century skills and higher-order thinking. In its design, it assists in the retention of knowledge for students but also supports real-life applications and graduated increases in problem complexity and critical thinking. This innovative teaching method introduces a fresh framework for mathematics education that is structured yet adaptable to diverse learning environments. The findings underscore the importance of integrating such inductive teaching methods in mathematics education, as they not only meet curricular standards but also equip students with the necessary skills to navigate the demands of a rapidly evolving world.

Keywords: *21st Century Skills, Exploratory learning, Higher-order thinking skills, Lesson exemplar, Mathematics, Teaching Pedagogy*

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INTRODUCTION

Traditionalists have long advocated for deductive teaching as the primary approach in mathematics education, citing its perceived advantages. However, research suggests that this method has significant limitations. Deductive teaching, characterized by a top-down approach where teachers present information for passive absorption by students, is often inadequate for fostering the interactive and dynamic skills required in the 21st century. Dreher et al. (2018) highlight the necessity for secondary mathematics teachers to possess a robust framework of content knowledge tailored to the specific challenges of teaching mathematics. Furthermore, González-Pérez and Ramírez-Montoya (2022) emphasize that traditional methods often lack active learning strategies necessary for developing complex reasoning and problem-solving skills. Their systematic review highlights a critical gap in pedagogical approaches, revealing that the rigid structure of deductive teaching fails to create environments conducive to the competencies demanded in modern education.

The limitations of deductive methods extend to professional development as well. Kim et al. (2019) argue for the importance of context-specific understanding in teaching practices, noting that traditional methods do not offer meaningful opportunities for educators' growth. The Teacher Instructional Practices and Processes System (TIPPS) reveals that deductive approaches lack reflective feedback, which is crucial for improving instructional quality, thus diminishing their effectiveness in promoting critical thinking and problem-solving. Additionally, Lavi et al. (2021) explore the perceptions of STEM alumni and students regarding the development of 21st-century skills through diverse teaching methods. Their findings suggest that insufficient training in innovative pedagogical strategies leaves many teachers dependent on outdated approaches, which fail to engage students meaningfully or foster essential skills.

To address these limitations and promote the development of 21st-century skills such as critical thinking and reasoning, an alternative approach is needed. Research has shown that inductive teaching can be more effective in promoting higher-order thinking skills and conceptual understanding in mathematics education (Harappa, 2022). Inductive teaching encourages students to actively construct knowledge by providing examples and allowing them to conclude (Harappa, 2022). Through the process of discovery and making connections between different mathematical concepts, students develop a deeper understanding of the subject matter (Pentang, 2021). Inductive teaching methods, which prioritize active student engagement and exploration, have been shown to foster deeper understanding and critical thinking. Mixed teaching approaches that blend student-centered and teacher-centered methods are crucial, as they align well with inductive teaching principles. These methods encourage students to draw conclusions and form hypotheses based on their experiences, enhancing retention and comprehension (Li et al., 2020).

Inductive teaching methods encourage students to derive mathematical concepts through exploration and guided inquiry. Szabo et al. (2020) stress the importance of teaching problem-solving to develop critical thinking skills. They advocate for Polya's heuristic, which aligns with inductive methods, creating an environment where students actively engage in problem-solving. The authors argue that this engagement cultivates independent thinkers who are better equipped to tackle real-world challenges, making inductive strategies more effective than deductive methods, which often rely on rote memorization and direct instruction.

Contextual learning through inductive teaching significantly boosts students' performance in dimensional mathematics by presenting them with real-world problems. Using authentic materials

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further enhances this effect by engaging learners more personally and motivating them. Teachers can improve their instructional competence by utilizing authentic materials to reinforce and expedite learning (Oclarit, Daguplo, & Casinillo, 2021). Reston (2015) highlighted the need for specialized professional development in the Philippines to address gaps in mathematics teachers' content knowledge and pedagogical approaches, with a focus on inductive teaching methods that promote active engagement and align with the K to 12 curriculum.

This project focuses on the development of PESCAD, an inductive mathematics teaching method designed to foster active student engagement, critical thinking, and conceptual understanding. The study explores PESCAD's effectiveness in developing 21st-century skills and higher-order thinking by analyzing its guiding steps, lesson exemplar format, and specific instructional procedures. It also details the development of the lesson exemplar, including its layout, procedural steps, and instructional use. Additionally, the project investigates how mathematics teachers validated PESCAD in terms of reliability, effectiveness, feasibility, inclusivity, and flexibility. Unlike traditional deductive approaches, which emphasize rote memorization and teacher-centered instruction, PESCAD offers a structured yet flexible framework that facilitates discovery, guided inquiry, and meaningful knowledge construction. By addressing key limitations of deductive methods—such as the lack of dynamic student interaction and opportunities for deeper learning—PESCAD promotes exploration, collaboration, and gradual mastery of complex mathematical concepts. Its design bridges the gap between theory and practice, providing an innovative framework that aligns with modern educational research advocating for strategies that prioritize problem-solving, reasoning, and intellectual autonomy.

This study uniquely contributes to the field of inductive teaching by introducing PESCAD, a method specifically designed for mathematics education. With a structured sequence of Problem, Examination, Solution, Checking, Appeals, and Discussion, PESCAD integrates problem-driven discovery with opportunities for reflection and collaborative learning. The Appeals and Discussion steps are particularly innovative, allowing students to critically evaluate their work and voice their misconceptions, which is rarely emphasized in other models. Furthermore, PESCAD bridges theoretical principles of inductive learning with practical application by aligning with DepEd's standards and the Most Essential Learning Competencies (MELCs), making it accessible and implementable in real-world classrooms. By addressing gaps in feedback mechanisms and offering a replicable structure for diverse mathematical topics, this study extends the potential of inductive teaching beyond traditional methods, making it a pioneering contribution to fostering critical thinking and 21st-century skills.

METHODOLOGY

Research Design

The research design employed in this study is developmental, to create and validate an inductive mathematics teaching method as a lesson exemplar. The research design was chosen to develop a lesson exemplar, specifically, the teaching method called PESCAD, and by gathering data from mathematics teachers at the basic education level, the material was validated. These teachers' perceptions of PESCAD formed the core data for the study.

The development of the PESCAD-incorporated lesson exemplar followed a step-by-step procedure guided by the modified ADDIE model (Analysis, Design, and Development). The analysis phase focused on comparing the benefits of inductive teaching to deductive teaching in developing 21st-century and higher-order thinking skills. The designing phase involved creating a guide for the steps of the PESCAD method, determining the format of the lesson exemplar, and selecting the lesson to be used. The development phase entailed designing the layout, specifying procedures for each step of the PESCAD method, and incorporating instructional and assessment activities. Finally, the validation phase involved assessing the PESCAD-incorporated lesson exemplar after its development.

Research Locale

The study was conducted at Nueva Ecija High School (NEHS) in Cabanatuan City, chosen for its pool of experienced mathematics teachers. These teachers, with firsthand experience in various teaching methods, are well-equipped to provide insights into the effectiveness and feasibility of the PESCAD method within the local educational context. Purposive sampling was used to select participants, ensuring that only experienced mathematics teachers with relevant knowledge and expertise were included. Additionally, five validators, including mathematics educators and experts in instructional strategies, were selected to provide informed feedback and enhance the study's validity and reliability.

Using a small, localized sample of validators, such as the experienced mathematics teachers from NEHS, presents potential limitations that could affect the generalizability of the findings. While purposive sampling ensures that participants have relevant knowledge and expertise, the sample may not fully represent the diverse range of educators and educational contexts across other regions or schools. The findings may be specific to the local educational environment, which could differ significantly from other areas in terms of curriculum, resources, teaching practices, and student needs.

Data Gathering Procedure

The initial phase involved a thorough analysis of the need for the PESCAD method in mathematics instruction. This was conducted through a review of existing literature about the challenges faced by teachers in teaching mathematics through the deductive teaching method. This analysis highlighted the necessity for a structured approach that enhances 21st-century skills and higher-order thinking among students, thereby informing the rationale for developing the PESCAD method.

In response to this need, the researcher structured an inductive teaching method called PESCAD, which stands for Problem-Examination-Solution-Checking-Appeals-Discussion. Each step of this method is designed to engage students in active learning. Once the structure of PESCAD was established, the researcher created a lesson exemplar based on this method. The topic, "Solving Quadratic Equations (through completing the square)" was selected, and deemed an effective example for incorporating PESCAD. The researcher referenced the Most Essential Learning Competencies (MELCs) to align the learning objectives and content of the discussion part within the lesson exemplar, ensuring relevance and compliance with educational standards.

The researcher developed a program of work to materialize PESCAD into a lesson exemplar, providing a clear structure for inductive teaching in mathematics, as assessed by basic education math teachers at Nueva Ecija High School. The lesson exemplar focused on the Grade 9 mathematics topic of "Solving Quadratic Equations (through completing the square)."The content and performance standards were aligned with the MELCs, ensuring that the lesson met educational requirements. The PROCEDURES section of the standard lesson exemplar format was modified to incorporate the steps of PESCAD, replacing the original parts while maintaining the integrity of the other sections. Additionally, annotations that serve as a guide for completing the lesson exemplar were revised to reflect the new procedures based on the PESCAD method.

To validate the findings, a panel of validators—comprised of experienced mathematics educators and instructional strategy experts—was convened. The self-made validating instrument demonstrated strong content and construct validity through expert reviews and alignment with established educational frameworks, while reliability testing revealed a Cronbach's Alpha coefficient of 0.87, indicating strong internal consistency. Selected for their relevant expertise and familiarity with the PESCAD method, the validators were presented with the analyzed data and findings. The feedback was crucial for validating the reliability and effectiveness of the PESCAD approach, ensuring that the study's conclusions were well-supported and reflective of the participants' authentic experiences.

Data Analysis

A literature review informed the foundational structure of PESCAD, aligning it with instructional best practices and addressing challenges identified in existing mathematics teaching methods. Throughout the study, decisions on refining PESCAD were data-driven, guided by participant feedback to ensure relevance and practicality.

The lesson exemplar was developed through reflective analysis, aligning its content with the PESCAD steps to enhance students' critical thinking and problem-solving skills. Quantitative data analysis included calculating weighted means and using a verbal description scale to gauge the effectiveness, feasibility, and inclusivity of PESCAD. Ranking prioritized participant responses, highlighting key strengths of the method.

Open-ended responses were transcribed, revealing participants' nuanced insights into the applicability and potential impact of PESCAD. This combination of methods provided a well-rounded, robust foundation for validating the effectiveness of the PESCAD method.

RESULTS AND DISCUSSION

Analysis of the Inductive Teaching Method

The analysis of teaching methods in mathematics reveals significant limitations in the deductive approach, particularly in its inability to foster higher-order thinking skills (HOTS) and critical reasoning. Students often struggle with abstract formulas, resorting to blind memorization, which hinders their ability to think critically and solve problems effectively (Prakash, 2022). This challenge is corroborated by Behar-Horenstein and Niu (2011), whose systematic review highlights the ineffectiveness of deductive methods in cultivating HOTS, suggesting that over-reliance on such methods yields inconsistent outcomes in developing critical thinking skills.

Evidence from Ichsan et al. (2019) further underscores the inadequacy of deductive teaching in preparing students for modern problem-solving tasks. The low HOTS scores observed across various educational levels demonstrate that traditional deductive approaches fail to equip students with the necessary cognitive tools for success in complex, real-world scenarios.

In contrast, inductive teaching methods have been shown to better support the development of critical thinking and problem-solving skills. For instance, Amer Atta et al. (2015) found that inductive approaches, being student-centric, allow learners to derive meaning from specific clues, fostering a deeper understanding and use of HOTS. The natural inclination of students to employ

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inductive reasoning, as observed by Charalambous et al. (2019), emphasizes the importance of integrating this method in classrooms to enhance problem-solving abilities and deepen mathematical understanding across educational levels.

Culturally relevant approaches, such as those grounded in ethnomathematics, further align with inductive principles. Imswatama and Lukman (2018) demonstrated that connecting mathematical concepts to students' cultural contexts not only improves engagement but also promotes effective problem-solving, making the learning process more meaningful and relatable. Similarly, Collins (2014) found that inductive teaching strategies, which challenge students to actively engage in higher-order thinking tasks, increase motivation, intellectual autonomy, and mastery.

Overall, the inductive teaching method provides a robust framework for fostering 21st-century skills. By centering learning on student-driven discovery, it enables learners to construct their understanding, thereby encouraging critical thinking and intellectual independence. This approach, rooted in natural cognitive processes and cultural relevance, offers a pathway to making mathematics education both effective and accessible, addressing the shortcomings of traditional deductive teaching methods.

Designing of PESCAD

The researcher structured an inductive method in mathematics teaching called PESCAD, which stands for Problem-Examination-Solution-Checking- Appeals-Discussion, which are steps of the said method. In the Problem part, the teacher will write a solved math problem on the board involving a lesson that is yet to be discussed. The students will then Examine or analyze the problem. After several minutes of Examination, the teacher will give another problem with the same case as the previous problem, but this time, students will find its Solution. When the students are done solving, the teacher will present the correct solution, and the students will check their work. The students will be given time to raise Appeals for their errors. These appeals from the students will be the basis of the Discussion of the concept.

After the structure was established, the researcher materialized this method as a lesson exemplar. The researcher obtained a template of the standard format of lesson exemplars from DepEd online. For the topic to be covered, the researcher chose "Solving Quadratic Equations (through completing the square)," deemed a good basic example for incorporating PESCAD. Most Essential Learning Competencies (MELCs) were also used as a reference for the learning objectives and the content of the discussion part in the lesson exemplar.

The design of PESCAD effectively translates the principles of inductive learning into a structured, interactive mathematics teaching method. Each guiding step of PESCAD—Problem, Examination, Solution, Checking, Appeals, and Discussion—provides students with opportunities to analyze, engage, and reflect, promoting active learning and deeper understanding. The development of the lesson exemplar, grounded in DepEd's standard template and aligned with MELCs, demonstrates PESCAD's practical applicability in teaching complex mathematical topics like quadratic equations. By systematically guiding students through problem-solving and critical reflection, PESCAD supports the cultivation of essential skills in mathematics, making it a promising method for fostering 21st-century competencies and higher-order thinking.

Development of PESCAD-Incorporated Lesson Exemplar

The researcher made a program of work in the development of PESCAD, which materialized as a lesson exemplar, which was meant to provide a clear structure of inductive teaching in mathematics as assessed by the basic education math teachers of NEHS.

The lesson exemplar comprised the topic in Grade 9 mathematics: "Solving Quadratic Equations (through completing the square)". The content and performance standards were all based on MELCs. The PROCEDURES section of the standard format of the lesson exemplar was modified. The original parts were replaced with the steps of PESCAD, which are the problem, examination, solution, checking, appeals, and discussion. The rest of the parts were not altered. The annotation that serves as a guide in accomplishing the lesson exemplar was also altered in favor of the new procedures based on the steps of PESCAD.

Problem. The learners will be shown a problem involving solving quadratic equations by completing the square. This problem has a complete solution. The teacher may present different approaches to the problem considering the different mathematical laws or properties.

The solution must be as detailed as possible. The teacher may give more than one problem with different complexities.

Figure 1:

Problem Section of PESCAD

A. PROBLEM	*The learners will be shown with a problem involving solving quadratic equations by competing the square. This problem has a complete solution.*	The solution must be as detailed as possible. You may give more than one problem and with different complexities.

		Guide in Preparing the Exemplar (This does not appear in the actual exemplar.)
	$x^2+4x-5=0$ $x^2+4x=5$ $x^2+4x+4=5+4$ $x^2+4x+4=9$ $(x+2)^2=9$ $\sqrt{(x+2)^2}=\pm\sqrt{9}$ $x+2=\pm 3$ $x_1+2=3$ $x_1=3-2$ $\boxed{x_1=1}$ $x_2+2=-3$ $x_2=-3-2$ $\boxed{x_2=-5}$	

The figure shows the Problem section of the lesson exemplar, where the solved mathematical problem that the learners have to examine is parked. It also shows the guidelines for conducting the activity.

Examination. The learners will be given five minutes to examine the given solution. They may use other learning resources, electronic and/or printed.

Allow the students to think critically on their own. Provide the most minimal assistance you can give, such as highlighting specific numbers or using lines and arrows.

Figure 2:
Examination Section of PESCAD

B. EXAMINATION	*The learners will be given 5 minutes to examine the given solution. They may use other learning resources, electronic and/or printed.* $x^2 + 4x - 5 = 0$ $x^2 + 4x = 5$ $x^2 + 4x + 4 = 5 + 4$ $x^2 + 4x + 4 = 9$ $(x+2)^2 = 9$ $\sqrt{(x+2)^2} = \pm\sqrt{9}$ $x + 2 = \pm 3$ $x_1 + 2 = 3$ $x_1 = 3 - 2$ $x_1 = 1$ $x_2 + 2 = -3$ $x_2 = -3 - 2$ $x_2 = -5$	Allow the students to think critically on their own. Provide the most minimal assistance you can give such as highlighting certain numbers or using lines and arrows.

The figure shows the Examination section of the lesson exemplar where the clues and hints to solving the problem are parked. It also shows the guidelines for conducting the activity, including the time limit.

Solution. The learners will be given another problem similar to the previous problem. This time, students will solve independently with guidance from the previous problem with a complete solution. They will be given 20 minutes to solve the problem. The first 10 minutes is for individual activity, and the last 10 minutes is for a group activity so they can share ideas.

The problem the learners will solve must be similar to the problem they examined. It will serve as their basis for solving the problem. The teacher may give more than one problem with different complexities.

Figure 3:
Solution Section of PESCAD

C. SOLUTION	*The learners will be given another problem similar to the previous problem. This time, students will solve on their own with the guidance from the previous problem with complete solution. They will be given 20 minutes to solve the problem.* $x^2 + 12x - 64 = 0$ *The first 10 minutes is for individual activity and the last ten minutes is for group activity so they can share ideas with one another*	The problem that will be solved by the learners must be similar to the problem they examined. It will serve as their basis to solve the problem. You may give more than one problem and with different complexities.

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The figure shows the Solution section of the lesson exemplar where the problem whose nature is similar to the examined solution is parked. It also shows the guidelines for conducting the activity, including the time limit.

Checking. After 20 minutes, the teacher will demonstrate the correct solution to the problem. The learners will check their solution based on the correct solution. Show only the solution and the corresponding annotations. This is to give space for questioning in the APPEALS part.

Figure 4:
Checking Section of PESCAD

D. CHECKING	*After 20 minutes, the teacher will demonstrate the correct solution to the problem. The learners will check their solution based on the correct solution.*	Show only the solution and the corresponding annotations. This is to give space for questioning in **APPEALS** part.

$x^2 + 12x - 64 = 0$ $x^2 + 12x = 64$	*Transpose the constant term to the right side of the equal sign.*
$x^2 + 12x + 36 = 64 + 36$	*Find the constant term for the quadratic expression on the left side so that it will become a perfect square. To do that, divide the numerical coefficient of the second term then raise to the power of two.*
$(x + 6)^2 = 100$	Express the quadratic expression as a square of binomial.
$\sqrt{(x + 6)^2} = \pm\sqrt{100}$ $x + 6 = \pm 10$	Extract the square root of the expressions on both sides.
$x_1 + 6 = 10$ $x_1 = 10 - 6$ $x_1 = 4$ $x_2 + 6 = -10$ $x_2 = -10 - 6$ $x_2 = -16$	Solve for the two roots.

The figure shows the Checking section of the lesson exemplar, where the solution of the problem that the students solved is parked. It also shows the solution commentary to explain how each part of the solution was done.

Appeals. The learners will re-examine their solution to the problem and their analysis of the first solution. They will be given 3 minutes to raise appeals and questions about their errors. Allow the students to practice their inquiry skills.

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Figure 5:
Appeals Section of PESCAD

E. APPEALS	*The learners will re-examine their solution to the problem and their analysis on the first solution. They will be given 3 minutes to raise appeals and questions on their errors.*	Allow the students to practice their inquiry skills.

The figure shows the Appeals section of the lesson exemplar, where the instructions on how to do the activity are parked. It also shows the guidelines for conducting the activity, including the time limit.

Discussion. After all appeals are addressed, the teacher will discuss the mathematics lesson, referring to a teaching guide. More examples must be shown, and real-life applications should be included if applicable.

Figure 6:
Discussion Section of PESCAD

F. DISCUSSION	**COMPLETING THE SQUARE** Extracting square roots and factoring are usually used to solve quadratic equations of the form $x^2 - c = 0$. If the factors of the quadratic expression of $ax^2 + bx + c = 0$ are determined, then it is more convenient to use factoring to solve it. Another method of solving quadratic equations is by completing the square. This method involves transforming the quadratic equation $ax^2 + bx + c = 0$ into the form square of binomial $(x - h)^2 = \mathbf{k}$, where **k ≥ 0**. $h = b/2$ To solve the quadratic equation $ax^2 + bx + c = 0$ by completing the square, the following steps can be followed: 1. Divide both sides of the equation by then simplify. 2. Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side. 3. Add the square of one-half of the coefficient of x on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial. 4. Express the perfect square trinomial on the left side of the equation as a square of a binomial. 5. Solve the resulting quadratic equation by extracting the square root. 6. Solve the resulting linear equations. 7. Check the solutions obtained against the original equation. **For example:** 1. Solve the quadratic equation $2x^2 + 8x - 10 = 0$ by completing the square. **Solution:**	Refer to the teaching guide. Make sure to add real-life application if applicable.

Divide both sides of the equation by 2, then simplify

$2x^2 + 8x - 10 = 0 \quad \rightarrow \quad \frac{2x^2}{2} + \frac{8x}{2} - \frac{10}{2} = \frac{0}{2}$

$x^2 + 4x - 5 = 0$

Add 5 to both sides of the equation and simplify.

$x^2 + 4x - 5 = 0 \quad \rightarrow \quad x^2 + 4x - 5 + 5 = 0 + 5$

$x^2 + 4x = 5$

Add to both sides of the equation the square of one-half of 4

$x^2 + 4x = 5 \quad \rightarrow \quad x^2 + 4x + 4 = 5 + 4$

$x^2 + 4x + 4 = 9$

Express $x^2 + 4x + 4$ as a square of binomial.

$x^2 + 4x + 4 = 9 \quad \rightarrow \quad \left(x + \frac{4}{2}\right)^2 = 9 \rightarrow (x + 2)^2 = 9$

Solve $(x + 2)^2 = 9$ by extracting the square root

$(x + 2)^2 = 9 \quad \rightarrow \quad \sqrt{(x + 2)^2} = \pm\sqrt{9}$ *Square both side and simplify*

$x + 2 = \pm 3$

Solve the resulting linear equations.

$x + 2 = 3$ $x = 3 - 2$ *Transpose +2 on the right side of the equation* $x = 1$ *Simplify*	$x + 2 = -3$ $x = -3 - 2$ *Transpose +2 on the right side of the equation* $x = -5$ *Simplify*

The figure shows the Discussion section of the lesson exemplar, where the elaboration of the content of the mathematics lesson is parked. It also shows the guidelines for conducting the activity.

The development of PESCAD as an inductive method of teaching mathematics provides a structured yet flexible approach to guiding students through complex problem-solving. By incorporating stages such as Problem, Examination, Solution, Checking, Appeals, and Discussion, PESCAD offers students a scaffold learning experience that moves from observation to critical analysis, application, and reflection. This carefully designed lesson exemplar, aligned with the Most Essential Learning Competencies (MELCs) and tailored for Grade 9 mathematics, ensures that students engage deeply with concepts like solving quadratic equations through completing the square. Each stage, accompanied by clear annotations and guidelines, fosters a supportive environment where students actively build their understanding, self-check their progress, and inquire about their learning process. By promoting these practices, PESCAD not only meets curricular standards but also enhances students' critical thinking and inquiry skills, making it a valuable framework for inductive mathematics instruction.

Validation of PESCAD

Potential of PESCAD to Develop 21st Century Skills

Table 1 shows the perception of mathematics teachers from NEHS towards the efficacy of PESCAD in developing 21st-century skills.

It can be gleaned from this table that statements 2, 10, and 11, which pertain to Creativity, Initiative, and Productivity, respectively, were the highest with a mean computed as 3.67 and verbally described as “Extremely Agree”. The respondents found those as the most possible for PESCAD to develop among learners. On the other hand, the respondents found statement 7, which pertains to Technology- Literacy, the least possible for PESCAD to develop among learners with a mean value of 3.00.

Overall, respondents rated PESCAD’s effectiveness in fostering 21st-century skills highly, with a mean score of 3.46, indicating an "Extremely Agree" consensus. A critical aspect of developing these skills through inductive teaching methods lies in assessing competencies effectively. Thornhill-Miller et al. (2023) highlight the importance of educational frameworks that integrate the 4Cs—critical thinking, communication, collaboration, and creativity—into curricula. Assessment practices that align with inductive methods, such as PESCAD, can strengthen these competencies by ensuring students are both engaged and able to demonstrate their skills in meaningful, practical contexts. These insights underscore PESCAD’s potential to equip students with essential skills to navigate the demands of the modern world effectively.

Table 1:
Potential of PESCAD to Develop 21st Century Skills

	WM	*VI*	*Rank*
1. PESCAD helps students figure things out for themselves when they don’t have a teacher at their disposal.	3.50	EA	4
2. PESCAD empowers students to see concepts in a different light, which leads to innovation.	3.67	EA	1

3. PESCAD gets students to work together, achieve compromises, and get the best possible results from solving a problem.	3.50	EA	4
4. PESCAD helps students to learn how to effectively convey ideas among different personality types.	3.33	EA	9
5. PESCAD teaches students how to separate fact from fiction	3.33	EA	9
6. PESCAD helps students identify publishing methods, outlets, and sources while distinguishing between the ones that are credible and the ones that aren't.	3.50	EA	4
7. PESCAD helps students to adapt to the world more effectively in terms of technology and machines.	3.00	A	12
8. PESCAD helps students to adapt to changing circumstances.	3.50	EA	4
9. PESCAD is a penchant for setting goals, walking a team through the steps required, and achieving those goals collaboratively.	3.33	EA	9
10. PESCAD requires students to be self-starters.	3.67	EA	1
11. PESCAD improves students' ability to complete work in an appropriate amount of time.	3.67	EA	1
12. PESCAD is done through the connections one person makes with others around them.	3.50	EA	4
Overall Weighted Mean	*3.46*	*EA*	

Legends: 3.25 – 4.00 Extremely Agree (EA); 2.50 – 3.24 Agree (A); 1.75 – 2.49 Disagree (D); 1.00 – 1.74 Extremely Disagree (ED)

Potential of PESCAD to Develop Higher-Order Thinking Skills

Table 2 shows the perception of mathematics teachers from NEHS towards the potential of PESCAD in developing higher-order thinking skills.

The table indicates that respondents view statement 3, which pertains to Evaluation, as the most likely higher-order thinking skill to be developed among learners, with a mean value of 3.83, interpreted as "Extremely Agree." This suggests that respondents believe PESCAD effectively encourages students to synthesize their learning and make informed evaluations of the material. Conversely, statements 1 and 2, which relate to Analysis and Synthesis, received the lowest ratings, with mean values of 3.50, indicating they are seen as less likely to be developed through PESCAD.

The criterion "PESCAD helps students to adapt to the world more effectively in terms of technology and machines," which received the lowest score (3.00) and was verbally interpreted as "Agree," suggests a potential gap in the method's integration of technological tools and digital literacy.

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This rating implies that while PESCAD may support critical thinking and collaboration, its ability to prepare students for technological adaptability is less pronounced. Future research could focus on enhancing PESCAD by incorporating more technology-driven activities, such as using digital tools for problem-solving, integrating simulations, or applying machine-based learning scenarios. Strengthening this aspect could better align PESCAD with the demands of an increasingly tech-driven world.

Overall, the respondents expressed strong agreement that PESCAD has the potential to foster higher-order thinking skills among learners, reflected in a mean score of 3.61. Chinedu et al. (2014) argue that higher-order thinking skills should be an integral part of teaching and learning, emphasizing the need for curriculum components that promote individual, cooperative, and creative problem-solving. This highlights the importance of developing teaching methods like PESCAD that not only facilitate the growth of higher-order thinking skills but also equip educators with the techniques necessary for effective instruction. Ultimately, PESCAD shows promise in enhancing students' critical thinking capabilities, preparing them for the complexities of modern learning environments.

Table 2:

Potential of PESCAD to Develop Higher-Order Thinking Skills

	WM	*VI*	*Rank*
1. PESCAD allows students to begin understanding the underlying structure of knowledge and also can distinguish between fact and opinion.	3.50	EA	2
2. PESCAD encourages students to move beyond relying on previously learned information or analyzing items that the teacher is giving to them.	3.50	EA	2
3. PESCAD encourages students to mentally assemble all they have learned to make informed and sound evaluations of the material.	3.83	EA	1
Overall Weighted Mean	*3.61*	*EA*	

Legends: 3.25 – 4.00 Extremely Agree (EA); 2.50 – 3.24 Agree (A); 1.75 – 2.49 Disagree (D); 1.00 – 1.74 Extremely Disagree (ED)

Initial Perception of Reliability, Effectiveness, Feasibility, Inclusivity, and Flexibility of PESCAD

Table 3 shows the validation of PESCAD's reliability, effectiveness, feasibility, inclusivity, and flexibility.

The table indicates that respondents "Extremely Agree" with statements regarding the reliability, effectiveness, feasibility, inclusivity, and flexibility of PESCAD, with mean values of 3.33, 3.83, 3.50, 3.33, and 3.33, respectively. This consensus suggests that validators believe PESCAD can consistently enhance mathematics teaching, effectively fulfill its intended purpose of improving math learning through inductive teaching, and be conveniently implemented in various educational contexts.

The research underscores the significance of teachers' subject matter knowledge (SMK) and their capacity to implement innovative practices, such as problem-posing. Professional development is crucial for enhancing teachers' pedagogical knowledge and ensuring the reliable application of these strategies (Lee et al., 2018). Additionally, ethnomathematics-based materials have been shown to improve student engagement and learning outcomes, further affirming their effectiveness and feasibility in classroom settings (Imswatama & Lukman, 2018). This supports the validators' belief that PESCAD can reliably enhance mathematics learning. Furthermore, Léon et al. (2017) note that teaching quality is a predictor of student engagement and achievement, highlighting the need for effective pedagogical methods that foster inclusivity.

The feasibility of integrating innovative strategies into existing curricula is vital for success. For example, online instructional modules have been demonstrated to improve students' understanding of mathematics, showcasing the flexibility of teaching methods in enhancing learning outcomes (Moradi et al., 2018).

Flexibility in teaching practices is essential for addressing the diverse needs of students. Understanding students' learning styles can guide educators in developing effective strategies that improve academic performance (Cardino & Cruz, 2020).

The strong agreement among respondents regarding the key attributes of PESCAD reinforces its potential as a valuable method in mathematics teaching. By focusing on reliability, effectiveness, feasibility, inclusivity, and flexibility, PESCAD can significantly contribute to enhancing student learning experiences and outcomes in mathematics.

Table 3:
Initial Perception of Reliability, Effectiveness, Feasibility, Inclusivity, and Flexibility of PESCAD

	WM	*VI*
1. PESCAD can work consistently well in the mathematics teaching	3.33	EA
2. PESCAD fulfills its specified function (improve math learning through inductive teaching)	3.83	EA
3. PESCAD is possible to do easily or conveniently.	3.50	EA
4. PESCAD includes or covers different learning preferences, learning styles, or learning status.	3.33	EA
5. PESCAD can work with other subjects in the basic education curriculum or even in higher education.	3.33	EA

Legends: 3.25 – 4.00 Extremely Agree (EA); 2.50 – 3.24 Agree (A); 1.75 – 2.49 Disagree (D); 1.00 – 1.74 Extremely Disagree (ED)

Enhancement of PESCAD Based on Validation Results

The comments and suggestions were given by respondents according to the general validation for further improvement of PESCAD. The data gathered informed the further improvement of PESCAD.

The effectiveness of PESCAD in fostering retention is highlighted by its ability to develop higher-order thinking skills, which encourages students to become self-starters, enhancing memory retention. A mathematics teacher emphasized its potential flexibility, suggesting that PESCAD could be applied effectively across other disciplines, as validated by respondents who "Extremely Agree" with its adaptability. Additionally, incorporating real-life applications into lessons was recommended to strengthen students' understanding and retention of mathematical concepts. To further enhance its impact, a master teacher proposed including examples with increasing complexity to encourage critical thinking, paired with extended problem-solving time for students.

To improve PESCAD, several actions were implemented based on feedback from educators. First, emphasis was placed on enhancing higher-order thinking skills, as highlighted by a mathematics teacher from NEHS, to foster self-directed learning and increase retention rates. Additionally, PESCAD's applicability in other disciplines will be explored, given its flexibility as agreed upon by respondents. Real-life applications were incorporated into the framework to make lessons more relatable and engaging for students. Lastly, the program included a wider variety of examples with increasing levels of complexity, encouraging deeper critical thinking while allowing more time for students to solve these challenging problems.

Based on the validation from mathematics teachers from NEHS, the researcher, therefore, claimed that PESCAD is effective, as perceived by the experts, in addressing the issue of low performance of high school students in mathematics due to frequent use of deductive teaching methods, which limits the development of all 21st-century skills, as well as higher-order thinking skills. This can be achieved through the kinds of questions that are asked, the types of activities available to students, and the products students create (Conklin, 2012). To do this, an inductive teaching method is one of the most effective methods. Compared to deductive teaching, the inductive teaching method provides more opportunities for learners to use higher-order thinking skills because it is student-centric. Students construct the meaning through the specifics or clues presented to them.

In summary, the inductive teaching method is found to be more effective than deductive teaching in mathematics. PESCAD addresses the challenges associated with traditional deductive teaching methods by shifting the focus from teacher-led instruction to student-centered discovery. Unlike deductive methods that often rely on rote memorization and direct instruction, PESCAD engages students actively in the learning process through structured steps: presenting solved problems, examining new problems, solving them independently, and participating in discussions to refine understanding. This approach mitigates the passive learning and lack of critical engagement typical of deductive teaching, fostering critical thinking, collaboration, and deeper conceptual understanding. To integrate PESCAD into existing curricula, educators can align its steps with lesson objectives and include it as part of formative assessments or hands-on activities. For instance, when teaching "Solving Quadratic Equations (through completing the square)," PESCAD can guide students to discover and internalize problem-solving techniques through active participation. Potential barriers to integration include resistance to changing traditional teaching practices and time constraints within rigid curricula. To overcome these barriers, professional development workshops can train teachers in the PESCAD method, emphasizing its alignment with the 21st-century skills emphasized in modern curricula. Additionally, schools can allocate flexible periods for exploratory learning, ensuring sufficient time for the method's iterative and reflective processes. By addressing these challenges and leveraging teacher training and curricular adjustments, PESCAD can be seamlessly integrated into educational systems, enhancing student outcomes and preparing learners for real-world problem-solving.

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CONCLUSIONS

PESCAD as an inductive mathematics teaching method effectively fosters the development of 21st-century skills and higher-order thinking skills in students. By shifting the focus from rote memorization to student-driven discovery, PESCAD enhances essential competencies such as problem-solving, critical thinking, and intellectual autonomy, which are crucial for success in today's academic and professional landscapes. The structured steps of PESCAD—Problem, Examination, Solution, Checking, Appeals, and Discussion—facilitate an engaging and interactive learning environment, promoting active student participation and deeper understanding of complex mathematical concepts. Furthermore, validation from mathematics teachers highlights PESCAD's reliability, effectiveness, feasibility, inclusivity, and flexibility, particularly in addressing the limitations of traditional deductive teaching methods.

Future research should prioritize implementing PESCAD in actual classroom settings to evaluate its practical effectiveness in fostering higher-order thinking and retention among students. Conducting longitudinal studies would provide insights into PESCAD's sustained impact on learning outcomes. Expanding its application to other topics in high school mathematics and to different subjects, such as science or language arts, could determine its versatility and adaptability across curricula. Additionally, exploring its applicability across educational levels, from elementary to tertiary education, would help assess its broader relevance. To address current limitations, future studies should involve diverse student populations and gather direct feedback from learners, complementing expert validation. Comparative studies with other teaching methods could further illuminate PESCAD's unique contributions and areas for refinement. By addressing these areas, researchers can enhance the robustness of PESCAD and its potential to transform teaching strategies in various educational contexts.

Based on the findings of this research, it is recommended that educational institutions incorporate the PESCAD framework into their mathematics curricula to enhance student engagement and promote the development of 21st-century skills and higher-order thinking. To ensure its effective implementation, specific strategies should be employed, such as integrating PESCAD-based lesson plans into existing curriculum guides and aligning these lessons with the Most Essential Learning Competencies (MELCs). Teacher training programs should emphasize the importance of inductive teaching methods, equipping educators with the knowledge and strategies necessary to implement PESCAD effectively in their classrooms. Workshops and hands-on sessions could allow teachers to practice creating PESCAD-driven lessons with a focus on active learning, problem-solving, and collaborative activities. Additionally, ongoing professional development opportunities, such as coaching sessions and peer collaborations, should be provided to support teachers in refining their pedagogical approaches and utilizing innovative resources that align with PESCAD's principles. Creating a supportive community of practice can also help teachers share best practices and troubleshoot challenges in real-time. Schools should integrate technologies and digital tools that facilitate exploration and collaboration, such as interactive problem-solving platforms or virtual whiteboards, which complement PESCAD's structure. By fostering a collaborative learning environment that prioritizes active student participation and critical reflection, schools can address the challenges posed by traditional deductive methods and ultimately improve student performance in mathematics. These implementation strategies will ensure that PESCAD becomes a sustainable and impactful part of mathematics education.

Conflict of Interest

I hereby declare that I do not have any personal conflict of interest that may arise from the application and submission of my research proposal.

I understand that I may be held accountable by the Wesleyan University – Philippines, Graduate School for any conflict of interest that I have intentionally concealed.

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